

Generic junction conditions in brane-world scenarios

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We present the generic junction conditions obeyed by a co-dimension one brane in an arbitrary background spacetime. As well as the usual Darmois-Israel junction conditions which relate the discontinuity in the extrinsic curvature to the energy-momentum tensor of matter which is localized to the brane, we point out that another condition must also be obeyed. This condition, which is the analogous to Newton's second law for a point particle, is trivially satisfied when Z_2 symmetry is enforced by hand, but in more general circumstances governs the evolution of the brane world-volume. As an illustration of its effect we compute the force on the brane due to a form field.

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I. INTRODUCTION

Motivated by developments in string theory interest has recently focused on the idea that the 4D universe that we see is an embedding of a 3-brane in higher dimensional space-time, the general concept being known as a brane-world. In the most popular of these models the universe is hyper-surface, hyper-brane or co-dimension one object in a 5-dimensional background [1,2]. The ordinary matter is localized as a distributional source, with an additional assumption that the extra dimension, which is also non compact, is Z_2 symmetric; both ideas being having phenomenological roots in string theory scenarios in the more formal contexts of D-branes [3] and orbifolds [4] respectively. An enormous amount of effort has been directed toward understanding how the gravity acts in a cosmological setting within the framework of these models using coordinate dependent [5,6] and coordinate independent [7] approaches. Central to all treatments of this problem are the junction conditions which must be imposed across the discontinuity induced by the brane.

Consider the general case of a p -brane (p is the number of spatial dimensions of the brane, which has a $(p+1)$ -dimensional world-volume) in an n -dimensional spacetime. The generic dynamical properties of such an object can be treated in a relatively straightforward way (see, for example, ref. [8]) so long as the evolution is 'passive', that is, it takes place in a background which is fixed and unaffected by the presence of the brane, as would be the case to a very good approximation when the relevant coupling (gravitational, electromagnetic, or other) are sufficiently weak. In contrast if the 'active' effect of such couplings is sufficiently strong, there will be awkward singularity problems involving divergences which require non-trivial regularisation procedures whenever the co-dimension, $n-p-1$, is equal to 2 or more, as in the recently clarified case [9] of a self-gravitating string (1-brane) in ordinary 4-dimensional spacetime.

The specific case of co-dimension one ($n=p+2$), the hyper-surface or hyper-brane which includes the case of a 3-brane in 5 dimensions, is much simpler, since it has the convenient feature that its 'active' effect on the background does not give rise to divergences and can be allowed for in straightforward way in terms of simple discontinuities in the relevant field gradients. Of most interest in the context of General Relativity is the discontinuity in the space-time metric $g_{\mu\nu}$ which has been discussed in many works. In this paper we follow the approach presented in ref. [10] based on a treatment pioneered by Darmois [11], and finally cast into a conveniently coordinate independent form in terms of the tensorially well defined second fundamental form (or extrinsic curvature tensor) by Israel [12] in what is commonly quoted as the definitive reference on this topic.

The present article has been motivated by the observation made in ref. [10] that these Darmois-Israel junction conditions are incomplete since they only prescribe the active source effect of the hyper-brane on the background, and do not include the equation governing the passive motion of the world-volume within the background. This passive equation of motion is often derived in particular coordinate systems for diverse applications (see, for example, ref. [13]) and in the specific case of Z_2 symmetry, often used in brane-world models, it is trivial [5]. The main result of this paper is to cast this equation, which is analogous to Newton's 2nd law, in a coordinate independent tensorial form in an essentially equivalent way to that done by Israel for the case of the active effect.

Such form for the passive effect has so far been available only for the weakly coupled limit, for which [8] it takes the simple form

$$\overline{T}^{\mu\nu} K_{\mu\nu} = f, \quad (1)$$

where $\overline{T}^{\mu\nu}$ is the energy-momentum density tensor for the matter and vacuum energy localized on the brane, $K_{\mu\nu}$ is the second fundamental form or extrinsic curvature tensor, and \overline{f} is the magnitude of any orthogonal force density due to external fields in the direction normal to the brane. Interestingly the form of (1) is that of Newton's 2nd law, $\overline{T}^{\mu\nu}$ playing the role of mass and $K_{\mu\nu}$ that of acceleration, and the more general form which we will derive here will just replace extrinsic curvature and the external force with their sectional averages over a small neighbourhood around the brane.

In fact (1) is a specialization of the more general equation $\overline{T}^{\mu\nu} K_{\mu\nu}{}^\rho = \perp^\rho{}_\nu f^\nu$, which applies to weakly coupled branes of any co-dimension [14], where $K_{\mu\nu}{}^\rho$ is the generalization of the second fundamental form and the f^ν is the external force density. Within this formalism $\eta_{\mu\nu}$ is the fundamental form (or metric) of the brane and its orthogonal complement is defined by $\perp_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. If one specializes to the co-dimension one case then the orthogonal projection can be expressed in terms of a single normal vector n_μ in the direction of the extra dimension; the orthogonal complement taking the form $\perp_{\mu\nu} = n_\mu n_\nu$. The orthogonal unit vector can also be used to specify the orthogonal force density magnitude, and the second fundamental tensor according to the prescriptions $\overline{f} = n_\nu \overline{f}^\nu$ and $K_{\mu\nu} = K_{\mu\nu}{}^\rho n_\rho$. Equivalently, without reference to the 3-index second fundamental tensor, the extrinsic curvature tensor can be expressed directly

$$K_{\mu\nu} = -\eta^\rho{}_\nu \overline{\nabla}_\mu n_\rho, \quad (2)$$

in terms of the tangentially projected differentiation operator, $\overline{\nabla}_\nu = \eta^\rho{}_\mu \nabla_\nu$. For the rest of this article we will work with the 2-index version of the second fundamental form as is sensible in the case of co-dimension one objects under consideration here. Nonetheless, we would like to point out that the notation and some of the methods we use are applicable more generally.

II. ACTIVE GRAVITATIONAL SOURCE EQUATION — THE DARMOIS-ISRAEL FORMULA

We start by introducing notation that is applicable to a generic field quantity Q say, before applying it to the specific case of the extrinsic curvature. The field is assumed to vary smoothly within a neighbourhood $\zeta_- < \zeta < \zeta_+$ of a timelike hypersurface that is identified as the world-volume of the brane under consideration. One can assume that the brane is situated at the locus $\zeta = 0$ with respect to coordinates chosen so that ζ measures the normal distance from the world-volume, and hence the unit normal will be given by $n_\nu = \partial_\nu \zeta / (\partial_\mu \zeta \partial^\mu \zeta)^{1/2}$. We are concerned with limit configurations for which the normal derivative $Q' = \partial Q / \partial \zeta = n^\nu \partial_\nu Q$ becomes extremely large compared with the corresponding tangential gradient components in the region under consideration. Therefore, the effective discontinuity as perceived on a scale large compared with the thickness $\zeta_+ - \zeta_-$ will be given by $[Q] = Q^+ - Q^-$. This discontinuity will also be expressible as $[Q] = \overline{Q}'$ where $\overline{\partial}_\nu \overline{Q} \simeq n_\nu [Q]$, using an overline for the sectional integral $\overline{Q}' = \int_{\zeta_-}^{\zeta_+} Q' d\zeta$. Conversely, this is equal to the coefficient of the Dirac distribution to which this derivative will tend in the ultra-thin limit, that is, $Q' = \overline{Q}' \delta[\zeta]$. If we now define $\langle Q \rangle = (Q^+ + Q^-)/2$, we see that the sectional integral of the product $Q Q'$ can be expressed as $\overline{Q Q'} = \langle Q \rangle [Q]$.

In such a thin brane limit, when the transverse derivatives are considered as negligibly small compared with normal ones, then according to the standard analysis [10] the dominant contribution to the normal derivative of the second fundamental form of the constant ζ hyper-surfaces will be given by

$$K'_{\mu\nu} \simeq \eta^\kappa{}_\mu \eta^\lambda{}_\nu \perp^\rho{}_\sigma \mathcal{R}_{\rho\kappa}{}^\sigma{}_\lambda, \quad (3)$$

where $\mathcal{R}_{\rho\mu}{}^\sigma{}_\nu$ are the components of the Riemann tensor of the n -dimensional background geometry and the symbol \simeq denotes equality to the dominant contribution. Contraction of this formula gives the relation $K' \simeq \perp^\rho{}_\sigma \mathcal{R}^\sigma{}_\rho$, where $K = K^\nu{}_\nu$ is the extrinsic curvature scalar and $\mathcal{R}_{\mu\nu} = \mathcal{R}_{\sigma\mu}{}^\sigma{}_\nu$ is the background Ricci tensor. Taking account of the fact that, even in the thin limit for which the background curvature becomes very large, the intrinsic curvature of the world-volume will remain relatively negligible, it can be seen that (3) also implies

$$K'_{\mu\nu} \simeq \eta^\kappa{}_\mu \eta^\lambda{}_\nu \mathcal{R}_{\kappa\lambda}. \quad (4)$$

Thus, one can deduce that the dominant components of the background Ricci tensor will be given by the asymptotic formula

$$\mathcal{R}_{\mu\nu} \simeq K'_{\mu\nu} + K' \perp_{\mu\nu}, \quad (5)$$

whose contraction gives the expression $\mathcal{R} \simeq 2K'$ for the background Ricci scalar $\mathcal{R} = \mathcal{R}^\nu{}_\nu$.

For physical applications one is usually concerned with the Einstein tensor $\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$ whose dominant components, which are exclusively tangential, are given by

$$\mathcal{G}_{\mu\nu} \simeq K'_{\mu\nu} - K'\eta_{\mu\nu}. \quad (6)$$

The active effect of the brane — the Darmois-Israel formula — can then be deduced by performing the sectional integral of this quantity. Using the notation conventions introduced above, one can deduce that

$$\bar{\mathcal{G}}_{\mu\nu} = [K_{\mu\nu}] - [K]\eta_{\mu\nu}. \quad (7)$$

In n -dimensions the Einstein equations take the form

$$\mathcal{G}_{\mu\nu} = (n-2)\Omega^{[n-2]}G_N^{[n]}T^{\mu\nu} - \Lambda^{[n]}g_{\mu\nu}, \quad (8)$$

where $\Omega^{[n-2]}$ denotes the surface area of the unit $(n-2)$ -sphere, $G_N^{[n]} = M^{2-n}$ is the n -dimensional analogy of the Newton's constant defined in terms of mass scale M , and $\Lambda^{[n]}$ is the background cosmological constant. In 4-dimensional spacetime, $\Omega^{[2]} = 4\pi$ and $M = M_{\text{Pl}}$ is the standard Planck mass, while in the 5-dimensional case of most interest here, $\Omega^{[3]} = 2\pi^2$ and M is the corresponding mass scale, which might be related to M_{Pl} in order to solve the hierarchy problem. By performing the relevant sectional integration, and substituting in (7) one can deduce the standard form of the Darmois-Israel conditions

$$[K_{\mu\nu}] = \frac{\Omega^{[n-2]}}{M^{n-2}} \left((n-2)\bar{T}_{\mu\nu} - \bar{T}^\rho{}_\rho \eta_{\mu\nu} \right), \quad (9)$$

for the gravitational effect of the sectionally integrated energy-momentum density tensor $\bar{T}^{\mu\nu} = \int_{\zeta_-}^{\zeta_+} T^{\mu\nu} d\zeta$.

III. PASSIVE WORLD-VOLUME EVOLUTION EQUATION

Having re-derived this well-known formula governing the ‘active’ gravitational source, we now turn our attention to the previously overlooked question of the corresponding equation governing the dynamical evolution of the brane world-volume, which can be obtained from the local dynamical equation

$$\nabla_\mu T^{\mu\nu} = f^\nu, \quad (10)$$

where f^ν is the force density resulting from coupling to external fields not taken into account in the evaluation of the locally concentrated energy-momentum density $T^{\mu\nu}$. The sectional integral of $T^{\mu\nu}$ gives the surface energy-momentum density $\bar{T}^{\mu\nu}$ which, in the ultra-thin limit, is given by $T^{\mu\nu} = \bar{T}^{\mu\nu} \delta[\zeta]$. It follows immediately from (10) that the orthogonal force density $f = n_\nu f^\nu$ will be given by

$$f = \nabla_\mu (T^{\mu\nu} n_\nu) - T^{\mu\nu} \nabla_\mu n_\nu, \quad (11)$$

via a standard manipulation.

Since the Einstein tensor (6) has a tangential form and is proportional to $T^{\mu\nu}$, it can be seen from (2) that the last term can be expressed as $T^{\mu\nu} \nabla_\mu n_\nu \simeq -T^{\mu\nu} K_{\mu\nu}$, while the first term will give a vanishing surface contribution by Green's theorem when we integrate to obtain the total sectional force density $\bar{f} = \int_-^+ f d\zeta$. This is given by

$$\bar{f} = \overline{T^{\mu\nu} K_{\mu\nu}}. \quad (12)$$

It can be seen from (7) that the dominant contribution to $T^{\mu\nu}$ is proportional to $(g^{\mu\rho}g^{\nu\sigma} - \eta^{\mu\nu}g^{\rho\sigma})K'_{\rho\sigma}$, so substituting $K_{\mu\nu}$ for Q in the general formula $\overline{Q'Q} = [Q]\langle Q \rangle$ we finally obtain the required discontinuous generalisation of (1) in the form

$$\overline{T}^{\mu\nu}\langle K_{\mu\nu}\rangle = \overline{f}, \quad (13)$$

where we recall that $\langle K_{\mu\nu}\rangle = \frac{1}{2}(K_{\mu\nu}^+ + K_{\mu\nu}^-)$ denotes the average of the values of the second fundamental form on the two sides.

To actually apply the evolution equation (13) it is necessary to know enough about the intrinsic mechanics of the brane to be able to evaluate its surface energy-momentum density $\overline{T}^{\mu\nu}$ and the force term \overline{f} , which will generally be attributable to the action of background fields whose energy-momentum density $T_{\text{ba}}^{\mu\nu}$ will provide a bounded contribution to the total stress energy density distribution $T_{\text{to}}^{\mu\nu} = T_{\text{ba}}^{\mu\nu} + \overline{T}^{\mu\nu} \delta[\zeta]$. The requirement that this total should satisfy the divergence condition $\nabla_\mu T_{\text{to}}^{\mu\nu} = 0$ implies by (10) that the force density term should be given by $f = n_\nu f^\nu$ with $f^\nu = -\nabla_\mu T_{\text{ba}}^{\mu\nu}$. In the ultra-thin limit this gives $f^\nu \simeq -n_\mu T_{\text{ba}}^{\mu\nu}$ and hence, the corresponding sectional integral is obtained as $\overline{f}^\nu = -n_\mu [T_{\text{ba}}^{\mu\nu}]$. Using this to evaluate the scalar force density $\overline{f} = n_\nu \overline{f}^\nu$, we finally obtain the world-volume evolution equation in the generically valid form

$$\overline{T}^{\mu\nu}\langle K_{\mu\nu}\rangle + [T_{\text{ba}}^{\mu\nu}] \perp_{\mu\nu} = 0. \quad (14)$$

IV. MINIMAL GAUGE COUPLING

We have already commented that Z_2 symmetry is often assumed in brane-world models motivated by the concept of orbifolds, and that the evolution equation which we have derived is trivially satisfied in this case. It is, however, interesting to consider what happens if this assumption is relaxed. A recent generalisation [15] envisages the spontaneous violation of the Z_2 reflection symmetry even without the intervention of an external force. Here, we consider the obvious further generalisation in which violation of reflection symmetry is not spontaneous, but imposed as a physical necessity by the presence of an external force of the kind that will arise from a minimal gauge field coupling.

The simplest kind of external field coupling that one might imagine is an antisymmetric r -form Ramond gauge field $A^{\{r\}}_{\nu_1 \dots \nu_r}$. For any such field there will be a corresponding gauge independent field defined as its exterior derivative by

$$F^{\{r+1\}}_{\nu_0 \nu_1 \dots \nu_r} = (r+1) \nabla_{[\nu_0} A^{\{r\}}_{\nu_1 \dots \nu_r]}, \quad (15)$$

where the square brackets denote antisymmetrization. The background action associated with such a field, will comprize of two terms, an external contribution — usually due to the kinetic energy term associated with the field — and coupling term, $\mathcal{I}_{\text{ba}} = \mathcal{I}_{\text{ex}} + \mathcal{I}_{\text{co}}$. One can write the external part in terms of an n -dimensional integral of a Lagrangian density $\mathcal{I}_{\text{ex}} = \int \mathcal{L}_{\text{ex}} \|g\|^{1/2} d^n x$ with the standard Lagrangian density taking the quadratic form

$$\mathcal{L}_{\text{ex}} = -\frac{m^{n-2-2r}}{2(r+1)!\Omega^{[n-2]}} F^{\{r+1\}}_{\nu_0 \dots \nu_r} F^{\{r+1\}}_{\nu_0 \dots \nu_r}, \quad (16)$$

where m is a fixed coupling constant having the dimensions of mass, whose presence can be seen to be redundant in the most familiar example, namely the case of Maxwellian electrodynamics characterised by $r=1$ with $n=4$, but not in more general situations. The corresponding external energy-momentum tensor $T_{\text{ex}}{}^\mu{}_\rho$ will be given by

$$T_{\text{ex}}{}^\mu{}_\rho = 2g_{\nu\rho} \frac{\partial \mathcal{L}_{\text{ex}}}{\partial g_{\mu\nu}} + g^\mu{}_\rho \mathcal{L}_{\text{ex}} = \frac{m^{n-2-2r}}{r!\Omega^{[n-2]}} \left(F^{\{r+1\}}_{\mu\nu_1 \dots \nu_r} F^{\{r+1\}}_{\rho\nu_1 \dots \nu_r} - \frac{1}{2(r+1)} g^\mu{}_\rho F^{\{r+1\}}_{\nu_0 \dots \nu_r} F^{\{r+1\}}_{\nu_0 \dots \nu_r} \right). \quad (17)$$

The standard coupling of such a field to a p -brane is using an action \mathcal{I}_{co} of the generalized Wess-Zumuno type

$$\mathcal{I}_{\text{co}} = \int \overline{\mathcal{L}_{\text{co}}} \|\gamma\|^{1/2} d^{p+1} \sigma, \quad (18)$$

which will require that $r = p+1$. The Lagrangian surface density has the form

$$\overline{\mathcal{L}_{\text{co}}} = \frac{1}{(p+1)!} \overline{\mathbf{j}}^{\{p+1\}}_{\nu_1 \dots \nu_{p+1}} A^{\{p+1\}}_{\nu_1 \dots \nu_{p+1}}, \quad (19)$$

with

$$\overline{\mathbf{j}}^{\{p+1\}}_{\nu_1 \dots \nu_{p+1}} = e^{\{p+1\}} \mathcal{E}^{\nu_1 \dots \nu_{p+1}}, \quad (20)$$

where $\mathcal{E}^{\nu_1 \dots \nu_{p+1}}$, normalized such that $\mathcal{E}^{\nu_1 \dots \nu_{p+1}} \mathcal{E}_{\nu_1 \dots \nu_{p+1}} = -(p+1)!$, is the usual antisymmetric surface element, $\|\gamma\|$ is the modulus of the determinant of the induced metric given with respect to internal coordinates σ^a ($a=1, \dots, p+1$) by $\gamma_{ab} = \eta_{\mu\nu} \partial_a x^\mu \partial_b x^\nu = g_{\mu\nu} \partial_a x^\mu \partial_b x^\nu$, and $e^{\{p+1\}}$ is a dimensionless charge coupling constant.

One can formally convert the world-volume integral (18) to the background spacetime integral

$$\mathcal{I}_{\text{co}} = \int \mathbf{j}^{\{p+1\} \nu_1 \dots \nu_{p+1}} A^{\{p+1\}}_{\nu_1 \dots \nu_{p+1}} \|\gamma\|^{1/2} d^n x, \quad (21)$$

by the introduction of a Dirac distribution with respect to the world-volume locus $x^\mu = \bar{x}^\mu \{\sigma\}$, and hence the source flux vector is given by

$$\mathbf{j}^{\{p+1\} \nu_1 \dots \nu_{p+1}} = \|\gamma\|^{-1/2} \int \delta^{(n)}[x - \bar{x}\{\sigma\}] \bar{\mathbf{j}}^{\{p+1\} \nu_1 \dots \nu_{p+1}} \|\gamma\|^{1/2} d^{p+1} \sigma. \quad (22)$$

The field equation can then be derived directly from this,

$$m^{n-2p-4} \nabla^\mu F^{\{p+2\}}_{\mu \nu_1 \dots \nu_{p+1}} = -\Omega^{[n-2]} \mathbf{j}^{\{p+1\}}_{\nu_1 \dots \nu_{p+1}}, \quad (23)$$

and (as shown elsewhere [14]) the corresponding generalised Faraday-Lorentz force density law is given by

$$f_\mu = \frac{1}{(p+1)!} F^{\{p+2\}}_{\mu \nu_1 \dots \nu_{p+1}} \mathbf{j}^{\{p+1\} \nu_1 \dots \nu_{p+1}}. \quad (24)$$

These expressions are valid for a general p -brane in an n -dimensional spacetime. For a co-dimension larger than one such a source gives rise to a divergence, for example the $n=4, d=2$ axion divergence in strings (see, for example, ref [16]), although this may be cancelled by the inclusion of an appropriately weighted dilaton field [17].

This kind of divergence problem does not arise in the co-dimension one case with which we are concerned here. In this case resulting field $F^{\{n\}}_{\nu_0 \dots \nu_{n-1}}$ will remain bounded, with just a discontinuity given according to (23), by the limit of the asymptotic formula $m^{-n} F^{\{n\}'}_{\mu \nu_1 \dots \nu_{n-1}} \simeq -n \Omega^{[n-2]} n_{[\mu} \mathbf{j}^{\{n-1\}}_{\nu_1 \dots \nu_{n-1}]}$. Computing the sectional integral, gives the following formula for the discontinuity

$$m^{-n} [F^{\{n\}}_{\mu \nu_1 \dots \nu_{n-1}}] = -n \Omega^{[n-2]} n_{[\mu} \bar{\mathbf{j}}^{\{n-1\}}_{\nu_1 \dots \nu_{n-1}]}, \quad (25)$$

with $\perp^\mu_{\nu_1} \bar{\mathbf{j}}^{\{n-1\} \nu_1 \dots \nu_{n-1}} = 0$. In the ultra-thin limit (22) can be replaced by the simpler formula

$$\mathbf{j}^{\{n-1\} \nu_1 \dots \nu_{n-1}} = \bar{\mathbf{j}}^{\{n-1\} \nu_1 \dots \nu_{n-1}} \delta[\zeta], \quad (26)$$

and it can be seen from (24) that the surface force density will be given in terms of the mean field $\langle F^{\{n\}}_{\nu_0 \dots \nu_{n-1}} \rangle$ by

$$\bar{f}_\mu = \frac{1}{(n-1)!} \langle F^{\{n\}}_{\mu \nu_1 \dots \nu_{n-1}} \rangle \bar{\mathbf{j}}^{\{n-1\} \nu_1 \dots \nu_{n-1}}. \quad (27)$$

Since there exists just one totally antisymmetric n -vector in an n -dimensional spacetime, one can write the corresponding physical field $F^{\{n\}}_{\nu_0 \dots \nu_{n-1}}$ in terms of a pseudo-scalar magnitude $F^{\{n\}}$ given by

$$F^{\{n\}}_{\nu_0 \dots \nu_{n-1}} = F^{\{n\}} \epsilon_{\nu_0 \dots \nu_{n-1}}, \quad (28)$$

where $\epsilon_{\nu_0 \dots \nu_{n-1}}$ is the background spacetime antisymmetric n -form normalized in an equivalent way to $\mathcal{E}^{\nu_1 \dots \nu_{p+1}}$. Therefore, the external energy-momentum tensor (17) will reduce to the form

$$T_{\text{ex}}{}^\mu{}_\rho = -\frac{1}{2m^n \Omega^{[n-2]}} (F^{\{n\}})^2 g^\mu{}_\rho, \quad (29)$$

since $\epsilon^{\mu \nu_1 \dots \nu_{n-1}} \mathcal{E}_{\rho \nu_1 \dots \nu_{n-1}} = -(n-1)! g^\mu{}_\rho$ and the field equation (23) will reduce to the form

$$\nabla_\mu F^{\{n\}} = \frac{m^n \Omega^{[n-2]}}{(n-1)!} \epsilon_{\mu \nu_1 \dots \nu_{n-1}} \mathbf{j}^{\{n-1\} \nu_1 \dots \nu_{n-1}}, \quad (30)$$

which implies that in a source free vacuum the value of the pseudo scalar $F^{\{n\}}$ will be constant, so that the effect of (29) will be perceived just as a positive increment to the cosmological constant. This increment will, however, be

subject to a discontinuous change across the brane world-volume, where it can be seen from the minimal coupling condition (20) that we shall have

$$[F^{\{n\}}] = e^{\{n-1\}} m^n \Omega^{[n-2]}, \quad (31)$$

subject to the understanding that the orientation of the brane is such that the unit normal is given by $n_\mu = \epsilon_{\mu\nu_1 \dots \nu_{n-1}} \mathcal{E}^{\nu_1 \dots \nu_{n-1}} / (n-1)!$.

The product $\|g\|^{1/2} \mathcal{L}_{\text{co}}$ is independent of the metric and so gives no contribution to the energy-momentum density, $T_{\text{co}}^{\mu\nu} = 0$ and hence that the background contribution in the world-volume evolution equation (14) will be given simply by $T_{\text{ba}}^{\mu\nu} = T_{\text{ex}}^{\mu\nu}$. Therefore, the required force density $\bar{f} = -n_\mu n_\nu [T_{\text{ba}}^{\mu\nu}]$ can thus be seen from (29) to be given by the constant value

$$\bar{f} = \frac{1}{2m^n \Omega^{[n-2]}} [(F^{\{n\}})^2] = e^{\{n-1\}} \langle F^{\{n\}} \rangle. \quad (32)$$

V. SUMMARY

We have derived in a coordinate independent way the passive effect of the world-volume evolution on a co-dimension one brane. This equation is interesting in the context of brane-world models when Z_2 symmetry is not imposed by hand and we have illustrated how such a possibility might manifest itself when the brane is minimally coupled to an $(n-1)$ -form field. We have showed that such a field is constant except for a discontinuity at the brane and, hence, the effective background cosmological constant will be different on either side of the brane giving us a realistic situation in which the models discussed in ref. [18] are valid. The formulae derived in this paper can be used to compute, for example, the evolution of the scale-factor in a FRW type model for the universe confined to a 3-brane in 5 spacetime dimensions [19].

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